Defects, Tunneling, and EPR Spectra of Single-Molecule Magnets

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ABSTRACT

We examine theoretically electron paramagnetic resonance (EPR) lineshapes as functions of resonance frequency, energy level, and temperature for single crystals of three different kinds of single-molecule nanomagnets (SMMs): Mn\textsubscript{12} acetate, Fe\textsubscript{8}Br, and the \( S = 9/2 \) Mn\textsubscript{4} compound. We use a density-matrix equation and consider distributions in the uniaxial (second-order) anisotropy parameter \( D \) and the \( g \) factor, caused by possible defects in the samples. Additionally, weak intermolecular exchange and electronic dipolar interactions are included in a mean-field approximation. Our calculated linewidths are in good agreement with experiments. We find that the distribution in \( D \) is common to the three examined single-molecule magnets. This could provide a basis for a proposed tunneling mechanism due to lattice defects or imperfections. We also find that weak intermolecular exchange and dipolar interactions are mainly responsible for the temperature dependence of the lineshapes for all three SMMs, and that the intermolecular exchange interaction is more significant for Mn\textsubscript{4} than for the other two SMMs. This finding is consistent with earlier experiments and suggests the role of spin-spin relaxation processes in the mechanism of magnetization tunneling.

INTRODUCTION

Single-molecule magnets (SMMs) have recently been the focus of much attention because of the possibility of macroscopic quantum tunneling of their magnetizations [1, 2] and possible applications in magnetic storage devices and quantum computers.
SMMs are composed of identical single-domain nanoscale molecules, comprised of a core of several transition-metal ions surrounded by many different species of atoms, and they have a large effective spin. The characteristics of SMMs are relatively weak exchange and dipolar interactions between molecules, a large zero-field energy barrier against magnetization reversal, and magnetization steps in their hysteresis loops, which indicate quantum tunneling despite the large spin values [4, 5, 6, 7, 8].

In this paper, we examine three different molecular magnets, which are briefly described in the following. The most extensively studied single-molecule magnet is $\text{[Mn}_{12}\text{O}_{12}(\text{CH}_3\text{COO})_{16}(\text{H}_2\text{O}_4)]\cdot2\text{CH}_3\text{COOH}\cdot4\text{H}_2\text{O}$ (abbreviated hereafter as Mn$_{12}$), which was first synthesized by Lis [9]. Each molecule in Mn$_{12}$ has an effective spin of $S = 10$. It is a uniaxial system with a zero-field energy barrier against magnetization reversal of 65 K [5, 6, 7]. Another well-studied magnet is $\text{[Fe}_{8}\text{O}_2(\text{OH})_{12}(\text{tacn})_6]\text{Br}_8\cdot9\text{H}_2\text{O}$ (abbreviated as Fe$_8$) [10]. Each molecule in Fe$_8$ also has a spin of $S = 10$ with a magnetization-reversal barrier of 30 K [11, 12]. This is a biaxial system so exhibits oscillations in tunneling rates with transverse magnetic field [13]. Recently synthesized is $\text{[Mn}_{4}\text{O}_{3}(\text{OSiMe})_3(\text{O}_2\text{CEt})_3(\text{dbm})_3]$ (abbreviated as Mn$_4$). Each molecule in Mn$_4$ consists of two kinds of Mn ions with mixed valence: three Mn$^{3+}$ ($S = 2$) and one Mn$^{4+}$ ($S = 3/2$) are located in a distorted cubane structure and antiferromagnetically coupled to give a ground-state spin of $S = 9/2$. The approximate magnetization-reversal energy barrier is 13 K in zero field [14]. All three of these SMMs show clear magnetization steps in hysteresis loops below their blocking temperatures.

Recently, electron paramagnetic resonance (EPR) experiments [12, 14, 15, 16] on single crystals of these SMMs showed interesting results for the lineshapes as a function of EPR resonance frequency, energy levels involved in the EPR transitions, and temperature. The temperatures employed are in the 2−50 K range, and the frequencies are in the 40−190 GHz range. We focus on the case when the magnetic field is applied along the easy axis at fixed frequency. In this case, the measured linewidths are much broader than the homogeneous line broadening caused by the lifetimes of the energy levels, and the linewidths increase as lower energy levels are involved with the EPR transitions. Additionally, some interesting features have been observed in the temperature dependence of the linewidths and lineshapes of the EPR spectra [6, 12, 14, 15, 16]. These temperature dependences were different for different molecular magnets. In this paper, we summarize our theoretical understanding of the linewidths’ dependence on frequency, energy level, and temperature for the three molecular magnets, Mn$_{12}$, Fe$_8$, and Mn$_4$. We also provide quantitative comparison of our calculated results with experimental data. The theoretical results on Mn$_4$ are preliminary and work on this system is still in progress.

**MODEL**

We start with a single-spin effective Hamiltonian considering embedded symmetry in each molecule and calculate the resonance linewidths using a density-matrix equation [17]. We find that an entirely single-spin picture cannot explain even qualitative trends in
the experimental data. Thus, it is inevitable to include many-body effects. As a simple
start, we assume that the second-order uniaxial anisotropy parameter, \( D \), and the \( g \)
factor may not be the same for all molecules. The microscopic origin of the distribution
in \( D \) has not yet been fully understood [18, 19]. In addition, each molecule may interact
with the rest of the molecules through exchange and/or dipolar interactions. Details of
the model and the technique are discussed in Refs. [15, 20, 21, 22]. This section is based
in part on our earlier work [15, 20].

Since each molecule in Mn\(_{12}\) has approximate S\(_4\) symmetry, the dominant single-spin
Hamiltonian reads, with applied magnetic field along the easy magnetization axis (z axis),
\[
\mathcal{H}_0 = -DS_z^2 - CS_z^4 - g\mu_B H S_z,
\]
(1)
where \( D = 0.55 \) K, \( C = 1.17 \times 10^{-3} \) K, and \( g = 1.94 \) [5]. The SMM Fe\(_8\) has approximate
D\(_2\) symmetry, so the Hamiltonian reads
\[
\mathcal{H}_0 = -DS_z^2 - E(S_x^2 - S_y^2) - g\mu_B H S_z,
\]
(2)
where \( D = 0.288 \) K, \( E = 0.043 \) K, and \( g = 2.0 \) [12]. In our calculations for Fe\(_8\), we
neglect the small \( E \) term because we are interested in the case with the applied field
along the easy axis. The SMM Mn\(_4\) has C\(_3\) symmetry, so the Hamiltonian reads
\[
\mathcal{H}_0 = -DS_z^2 - CS_z^4 - g\mu_B H S_z,
\]
(3)
where \( D = 0.632 \) K, \( C = 3.12 \times 10^{-3} \) K, and \( g = 2.0 \) [14].

We now introduce an interaction \( V(t) \) between the spin system and an oscillating
transverse magnetic field \( H_x \) with angular frequency \( \omega \equiv 2\pi\nu \). The interaction of the spin
system with the environment is governed by a time dependent density-matrix equation
[17, 23]. The power absorption between the energy levels \( M_s \) and \( M_s - 1 \) from the
oscillating field, up to first order in \( V_0 \) and near resonance (with a fixed value of \( D \), is
written as
\[
\frac{d\mathcal{E}}{dt} = \frac{(\mathcal{E}_{M,s-1} - \mathcal{E}_{M,s})}{\hbar^2} |\langle M_s|V_0|M_s-1\rangle|^2 \Delta(H)(\rho_{M,s} - \rho_{M,s-1}),
\]
(4)
\[
\Delta(H) \equiv \frac{\hbar^2(\gamma_{M,s-1,M,s})^2}{(g\mu_B)^2(H - H_{res})^2 + (\hbar\gamma_{M,s-1,M,s})^2},
\]
(5)
\[
H_{res} = \frac{\hbar\omega - D(2M_s - 1) - C(4M_s^3 - 6M_s^2 + 4M_s - 1)}{g\mu_B},
\]
(6)
where \( V_0 \) is the strength of the interaction \( V(t) \), \( \mathcal{E}_{M,s} \) is the energy of the level \( M_s \),
\( \rho_{M,s} \) is the population of the level \( M_s \), \( \Delta(H) \) is a Lorentzian lineshape function, and
\( H_{res} \) is the resonance field for Mn\(_{12}\) and Mn\(_4\). (For Fe\(_8\) there is no \( C \) term.) The ratio
\( \hbar\gamma_{M,s-1,M,s}/g\mu_B \) gives a linewidth due to the finite lifetime of any excited state. The
linewidths determined by \( \gamma_{M,s-1,M,s} \) are on the order of several to several tens of gauss
at the measured temperature (10 K for Fe\(_8\) and Mn\(_4\) and 25 K for Mn\(_{12}\), and they
decrease with increasing $M_s$. (In our convention, $M_s = +10$ is the ground state with $H > 0$.) However, the measured linewidths increase with increasing $M_s$, and the order of magnitude of the widths is a couple of hundred to a thousand gauss. Additionally, the dramatic temperature dependence of the linewidths cannot be explained by homogeneous broadening alone.

To resolve this discrepancy, we include distributions in $D$ and $g$, (called $D$-strain and $g$-strain effects, respectively) and intermolecular exchange and dipolar interactions. With Gaussian distributions in $D$ and $g$, centered at the experimentally determined values and with small standard deviations $\sigma_D$ and $\sigma_g$, we can average the power absorption (equation 4) over the distributions. To calculate the effect on the linewidths of the intermolecular exchange and dipolar interactions with fixed $D$ and $g$, we use a multispin Hamiltonian [20, 21, 22]:

$$H_{\text{tot}} = \sum_i \left[ H_{0i} + V_i(t) \right] + H^{(1)}, \quad H^{(1)} \equiv H^{\text{dipole}} + H^{\text{exch}},$$

where $H^{\text{dipole}}$ is the dipolar interaction between the molecules, and $\zeta_{jk}$ are the direction cosines of the vector between molecules $j$ and $k$ ($\vec{r}_{jk}$) relative to the easy axis ($z$ axis). The sum $\sum'_{jk}$ runs over all molecules, so that any two indices are not the same. $H^{\text{exch}}$ is the isotropic exchange interaction between the spins of nearest-neighbor molecules, where the exchange coupling constant $J_{ij}$ is $J$ if the $i$th and $j$th spins are nearest neighbors and zero otherwise. Assuming that $\sum_i V_i(t)$ is much smaller than $H^{(1)}$, which again is much smaller than $\sum_i H_{0i}$, we neglect $\sum_i V_i(t)$ and treat $H^{(1)}$ perturbatively. In a mean-field approximation, the sums of $A_{jk}$ and $J_{jk}$ (equations 9 and 10) can be separated from the spin operators.

Using the multispin Hamiltonian, we construct a probability density function for the resonance field. To compare with the measured linewidths, we calculate the second central moment of the resonance field, $\langle (H - \langle H \rangle)^2 \rangle$, to zero order in $H^{(1)}/k_BT$. To explain the experimental data, we need to vary two parameters for the $D$-strain and $g$-strain effects, and to vary three parameters [20] for the contribution of the spin-spin interactions: $\sum_{ij} J_{ij}^2$, $\Gamma \equiv \sum_{ij} A_{ij}^2$, and $\Lambda \equiv \sum_{ij} J_{ij} A_{ij}$.

**RESULTS AND DISCUSSION**

We find that the uniaxial anisotropy parameter $D$ is randomly distributed in all
three of the molecular magnets. This may be due to possible defects or imperfections in the samples. For Mn$_{12}$ and Mn$_4$, the distribution in the $g$ factor is also important, although this effect is not significant for Fe$_8$. The spreads in $D$ and $g$ vary from sample to sample by 30–40%. The effect of the distribution in $D$ on inhomogeneous broadening was observed in terahertz spectroscopy for Mn$_{12}$ [24]. The temperature dependence of the linewidths is mainly caused by the spin-spin interactions. The dipolar field was measured in a millimeter-wave study of Fe$_8$ [25]. For Mn$_4$, the intermolecular exchange interaction is stronger than the dipolar interaction, which is consistent with experiments on Mn$_4$ dimers [26] and monomers [27, 28]. For clarity, we discuss each SMM separately.

**Fe$_8$**

We find that for Fe$_8$ the distribution in $D$ and the intermolecular exchange and dipolar interactions contribute to the linewidths. The distribution in $D$ makes the linewidth increase with increasing energy level $M_s$, which can be seen from the resonance-field expression (equation 6). The intermolecular exchange and dipolar interactions make the linewidths increase with increasing $M_s$ and with decreasing the resonance frequency $\nu$. The reason is that the resonance field decreases with increasing $M_s$ and with decreasing $\nu$, which can be understood by equation 6. Combining the effect of the distribution in $D$ with the exchange and dipolar interactions explains well the trend and the magnitude of the measured linewidths as functions of frequency and energy level [15].

Next we discuss the temperature dependence of the linewidths. The distribution in $D$ alone cannot explain the interesting measured temperature dependence of the linewidths shown as symbols in figure 1 (a). The reason is that the linewidths caused by the distribution in $D$ are only slightly temperature dependent due to the temperature dependence of the natural linewidths $h\gamma_{M_{s-1,M_s}}/g\mu_B$ in equation 5. So this weak temperature dependence is monotonic and noticeable only for the small $M_s$ (at most about 100 G at 30 K for $M_s = 3$) [20]. However, the linewidths caused solely by the exchange and dipolar interactions at fixed $D$ and $g$ vary significantly with temperature, as shown in figure 1 (b). For the ground state $M_s = 10$, the linewidths decrease with increasing temperature in the whole temperature range. For $M_s = 9, 8,$ and $7$, the widths first increase sharply with temperature at low temperatures, and then decrease slowly with temperature at high temperatures. For $M_s = 6, 5, 4,$ and $3$, the widths increase with increasing temperature in the whole range. This trend was also seen in the experimental linewidths, confirming that the exchange and dipolar interactions are crucial to understanding the temperature dependence of the linewidths. The calculated linewidths including the distribution in $D$ and the spin-spin interactions are shown as curves in figure 1 (a). Here we use $\sigma_D \sim 0.0064 D$, $J = -7$ G, $\Gamma = 86$ G$^2$, and $\Lambda = -156$ G$^2$. The calculated linewidths agree with the measured linewidths except in the low-temperature range for large $M_s$ transitions ($M_s = 10, 9, 8$). At present, we do not fully understand this discrepancy. We note that the quality of the fit that includes a small ferromagnetic exchange interaction is comparable with that without exchange interaction, because the exchange coupling
Figure 1: (a) Calculated (curves) and measured (symbols) FWHM vs temperature at $\nu = 116.9$ GHz for Fe$_8$. Here the standard deviation of $D$, $\sigma_D \sim 0.0064D$, the exchange constant $J = -7$ G, $\Gamma = \sum_i A_{ij}^2/N = 86$ G$^2$, and $\Lambda = \sum_i J_{ij} A_{ij}/N = -156$ G$^2$. The solid curves, from bottom to top, correspond to $M_s = 3, 4, ..., 9, 10$. (b) Calculated FWHM caused by the exchange and dipolar interactions only, shown vs temperature at $\nu = 116$ GHz for Fe$_8$. Here $J = -7$ G, $\Gamma = 86$ G$^2$, and $\Lambda = -156$ G$^2$. Cited from Ref. [20].

constant is small compared with the linewidths[29].

**Mn$_{12}$**

We find that the inhomogeneous line broadening for Mn$_{12}$ is due to the distributions in $D$ and $g$, and to the dipolar interactions. The distribution in $D$ contributes to the variation of the linewidth as a function of $M_s$ and $\nu$ in the same way as for Fe$_8$. The distribution in $g$ makes the linewidths decrease with increasing $M_s$ and decreasing $\nu$, which is opposite to the effect of the distribution in $D$. The reason is that the resonance field decreases with increasing $M_s$ and decreasing $\nu$. The temperature dependence of the linewidths caused by only the distributions in $D$ and $g$ is very weak, so that it cannot explain the measured linewidths shown as symbols in figure 2 (a), which is similar to those for Fe$_8$. The contribution of the dipolar interactions to the linewidths is shown as a function of temperature in figure 2 (b). Unlike for Fe$_8$, the $M_s$ dependence of the dipolar broadening does not decrease with increasing temperature (the curves are almost parallel as the temperature increases). Combining the three effects (distributions in $D$ and $g$, and dipolar interactions), we find that the calculated linewidths agree well with the experimental data with $\sigma_D \sim 0.018D$, $\sigma_g \sim 0.002g$, and $\Gamma = 203$ G$^2$, as shown in figure 2 (a). Compared to the linewidths for Fe$_8$, the relatively weak temperature dependence for Mn$_{12}$ indicates that for Mn$_{12}$ the dipolar broadening is overshadowed...
by the effect of the distribution in $D$, which is three times as wide as for Fe$_8$. With the same parameter values, we also check that the $M_s$ and frequency dependence of the measured linewidths can be explained.

**Mn$_4$**

We find that for Mn$_4$ all of our considerations of the many-body effects - the distributions in $D$ and $g$, and the intermolecular exchange and dipolar interactions - substantially affect the inhomogeneous broadening. One minor difference of this SMM from the other two SMMs is that the effect of the distribution in $D$ disappears in the transition $M_s = 1/2 \rightarrow -1/2$ because the resonance field (equation 6) does not depend on $D$ at this transition. To examine the temperature dependence of the linewidths caused solely by $D$-strain and $g$-strain, we used the same spin-phonon coupling parameters as for Mn$_{12}$[23] to obtain the order of magnitude of the lifetimes of the energy levels. As expected from the analysis of the two other SMMs, we also find that for Mn$_4$ the distributions in $D$ and $g$ do not significantly change the dependence of the linewidths on temperature.

Figure 3 (a) shows the calculated linewidths caused by the exchange and dipolar interactions only at fixed $D$ and $g$ versus temperature at $\nu = 138$ GHz. Here we use the exchange coupling constant $J = -40$ G, two nearest neighbors along the easy axis (because the nearest-neighbor distance along this direction is half of the nearest-
Figure 3: (a) Calculated linewidths due to the exchange and dipolar interactions only vs temperature at $\nu = 138$ GHz for Mn$_4$. Here we use the exchange coupling constant $J = -40$ G, $\Lambda = \sum_{ij} J_{ij} A_{ij}/N = -1854.8$ G$^2$, and $\Gamma = \sum_{ij} A_{ij}^2/N = 344$ G$^2$. (b) The same linewidths as in (a) when the sign of $J$ is flipped and the other parameter values are kept the same.

neighbor distance perpendicular to this direction), $\Lambda = \sum_{ij} J_{ij} A_{ij}/N = -1854.8$ G$^2$, and $\Gamma = \sum_{ij} A_{ij}^2/N = 344$ G$^2$. The negative sign in $J$ means ferromagnetically coupled spins. If we assume the approximate lattice geometry as a tetragonal lattice and take the easy axis as the crystal $c$ axis, then we know that the sign of $\Lambda$ should be the same as that of $J$. We explore some other possibilities of combining the exchange with the dipolar interaction. For example, figure 3 (b) shows the linewidths due to positive $J$ (antiferromagnetically coupled) and the dipolar interaction. With positive $J$, the linewidth for $M_s = 9/2$ is greatly reduced and the linewidth for $M_s = 1/2$ is appreciably enhanced. This feature is quite different from the broadening with negative $J$.

Figure 4 shows the calculated linewidths versus temperature at $\nu = 138$ GHz. Here we use $\sigma_D = 0.01D$, $\sigma_g = 0.004g$, $J = -40$ G, $\Lambda = -1854.8$ G$^2$, and $\Gamma = 344$ G$^2$. The calculated linewidths agree reasonably with the experimental data except for the transition $M_s = 1/2 \rightarrow -1/2$. Notice that the magnitudes of the values of $J$ and $\Lambda$ are greatly enhanced compared to those for Fe$_8$. This suggests that Mn$_4$ has a stronger exchange interaction between molecules than Fe$_8$. Our value for $J$ is different by a factor of 2 from the experimentally extracted value, $J = -74$ G [27]. Therefore, our finding may support the proposed mechanism of spin-spin cross relaxation in Mn$_4$ [27].

With the same parameter values, we can explain the frequency dependence of the linewidths at $T = 10$ K shown in figure 5. Although there are substantial distributions in $D$ and $g$, the widths are somewhat dependent on the frequency, which indicates that the spin-spin interactions are not completely overshadowed by the $D$-strain and $g$-strain effects as in Mn$_{12}$. But definitely the frequency dependence is stronger than for Mn$_{12}$.
Figure 4: Calculated (curves) and measured (symbols) FWHM vs temperature at $\nu = 138$ GHz for Mn$_4$. Here we use $\sigma_D = 0.01D$, $\sigma_g = 0.004g$, $J = -40$ G, $\Lambda = \sum_{ij} J_{ij}A_{ij}/N = -1854.8$ G$^2$, and $\Gamma = \sum_{ij} A_{ij}^2/N = 344$ G$^2$. The solid curves, from bottom to top, correspond to $M_s = 1/2, 3/2, ..., 9/2$.

but weaker than for Fe$_8$.

CONCLUSIONS

We investigated the EPR lineshapes as functions of resonance frequency $\nu$, energy level $M_s$, and temperature $T$ for the three different molecular magnets, Mn$_{12}$, Fe$_8$, and Mn$_4$, when the magnetic field is applied along the easy axis and the frequency is kept fixed. In our calculations, intermolecular exchange and dipolar interactions were included, as well as distributions in the uniaxial anisotropy parameter $D$ and the $g$ factor. We find that the distribution in $D$ is present in all three SMMs but that the effect is strongest in Mn$_{12}$, and that the distribution in $g$ contributes to the linewidths for Mn$_{12}$ and Mn$_4$. The spin-spin interactions are responsible for the interesting trend of the temperature dependent lineshapes for all three magnets. For Mn$_4$, the exchange interaction is stronger than the dipolar interaction, which supports the conclusions derived from the earlier measurements[27]. The linewidths for the three magnets reveal different $M_s$, $\nu$, and $T$ dependences because of different contributions from the effects we are considering. Observation of EPR lineshapes could indicate which effects are dominant in a particular SMM, so that we may identify which mechanism leads to quantum tunneling.

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Figure 5: Calculated (curves) and measured (symbols) FWHM vs energy level $M_s$ at $T = 10$ K for $\nu = 72, 88, 117, 126, 138, \text{and} 146$ GHz for Mn$_4$. Here the values of $\sigma_D$, $\sigma_g$, $J$, $\Gamma$, and $\Lambda$ are the same as those in figure 4.

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REFERENCES


